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Effective $\bar{K}N$ interaction and role of chiral symmetry

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We study the consequence of chiral SU(3) symmetry in the kaon-nucleon phenomenology, by deriving the effective single-channel $\bar{K}N$ potential. It turns out that the $\pi\Sigma$ interaction is strongly attractive and plays an important role for the structure of the $\Lambda(1405)$ resonance. We discuss the implication of effective potential for the few-body kaonic nuclei.

Keywords: Chiral SU(3) dynamics; $\Lambda(1405)$ resonance; Kaonic nuclei.

1. Introduction

Physics of $\bar{K}$ in nuclei is now lively discussed, stimulated by the proposal of deeply bound kaonic nuclei.[1] Experimental searches for the simplest K^-pp system have reported a broad bump structure in the ΛN invariant mass spectrum, while the interpretation of the observed structure requires an elaborate analysis.[2] Rigorous few-body calculations with realistic potentials for the K^-pp system were presented in Refs. 3, indicating the bound K^-pp system above $\pi\Sigma N$ threshold.

The relevant energy region for the study of kaonic nuclei is far below the $\bar{K}N$ threshold, where the amplitude is dominated by the $\Lambda(1405)$ resonance. This means that the extrapolation of the $\bar{K}N$ interaction should be carefully performed with a proper treatment of the $\Lambda(1405)$. In this study, we rely upon the following guiding principles for the description of the $\bar{K}N$ scattering: chiral symmetry and coupled-channel dynamics. Chiral symmetry of QCD determines the low energy interaction between the pseudoscalar meson (the Nambu-Goldstone boson) and any target hadron,[4] and the importance of the coupled-channel dynamics has been emphasized in the phenomenological study of $\bar{K}N$ scattering, based on the strong attraction

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in this channel.[5] Theoretical framework based on these principles has been developed as the chiral coupled-channel approach, reproducing successfully the $\bar{K}N$ scattering data and the properties of the $\Lambda(1405)$ resonance.[6] Here we derive an effective $\bar{K}N$ interaction based on chiral dynamics,[7] and discuss its phenomenological consequence in the study of K^-pp system.[8]

2. Chiral interaction and coupled-channel approach

In the leading order of chiral perturbation theory, meson-baryon s-wave interaction at total energy $\sqrt{s}$ from channel j to i reads

$$V_{ij}(\sqrt{s}) = -\frac{C_{ij}}{4f^2}(2\sqrt{s} - M_i - M_j)\sqrt{\frac{E_i + M_i}{2M_i}}\sqrt{\frac{E_j + M_j}{2M_j}}, \qquad (1)$$

where f is the pseudoscalar meson decay constant, M_i and E_i are the mass and the energy of the baryon in channel i, respectively. The coupling strengths C_{ij} are collected in the matrix

$$C_{ij}^{I=0} = \begin{pmatrix} 3 & -\sqrt{\frac{3}{2}} & \frac{3}{\sqrt{2}} & 0 \\ & 4 & 0 & \sqrt{\frac{3}{2}} \\ & & 0 & -\frac{3}{\sqrt{2}} \\ & & & 3 \end{pmatrix},$$

for the $S = -1$ and $I = 0$ channels in the following order : $\bar{K}N$, $\pi\Sigma$, $\eta\Lambda$, and $K\Xi$. The properties of the interaction—sign, strength, and energy dependence—are strictly governed by the chiral low energy theorem. One observes that the interactions in *both* $\bar{K}N$ and $\pi\Sigma$ channels are attractive, which is inevitable as far as we respect chiral symmetry. As we will see below, these attractive forces are so strong that pole singularities of the amplitude are generated for both channels.

Since the system is strongly interacting, we need to perform nonperturbative resummation. In Refs. 6,7 this has been achieved by solving the Bethe-Salpeter equation

$$T_{ij}(\sqrt{s}) = V_{ij}(\sqrt{s}) + V_{il}(\sqrt{s})\, G_l(\sqrt{s})\, T_{lj}(\sqrt{s}), \qquad (2)$$

with the interaction kernel V_{ij} in Eq. (1) and the meson-baryon loop integral G_i in dimensional regularization. The solution of Eq. (2) is given in matrix form by $T = [V^{-1} - G]^{-1}$ under the on-shell factorization. The equivalent amplitude is also obtained in the N/D method, which guarantees the unitarity of the scattering amplitude.

3. Structure of the $\Lambda(1405)$ resonance

It has been shown that the chiral coupled-channel approach reproduces the experimental observables of $\bar{K}N$ scattering very well, generating the $\Lambda(1405)$ resonance dynamically.[6,9] The $\Lambda(1405)$ in this approach is naively interpreted as quasibound meson-baryon molecule. Indeed, several recent analyses support the meson-baryon molecular picture.[10]

In the present context, it is important to focus on the pole structure. In Ref. 11, it is found that the $\Lambda(1405)$ resonance is associated by two poles. Using the model given in Refs. 9, the poles are found at

$$z_1 = 1428 - 17i \text{ MeV}, \quad z_2 = 1400 - 76i \text{ MeV},$$

which appear above $\pi\Sigma$ threshold and below $\bar{K}N$ threshold. Since the two poles are located close to each other, the observed spectrum exhibits only one bump structure, which was interpreted as a single resonance, the $\Lambda(1405)$. The coupling strengths of the poles to the $\pi\Sigma$ and $\bar{K}N$ channels are different from one to the other. Therefore, these poles contribute to the $\bar{K}N$ and $\pi\Sigma$ amplitudes with different weights, leading to the different spectral shapes of two amplitudes.[11]

In order to study the origin of this interesting structure, we perform the resummation of the single-channel interaction by switching off the couplings to the other channels. This single channel $\bar{K}N$ interaction generates a relatively weak bound state below threshold, while the $\pi\Sigma$ amplitude exhibits a broad resonance above threshold:

$$z_1(\bar{K}N \text{ only}) = 1427 \text{ MeV}, \quad z_2(\pi\Sigma \text{ only}) = 1388 - 96i \text{ MeV}.$$

Thus, the attractive forces in diagonal $\bar{K}N$ and $\pi\Sigma$ channels already generate two poles between thresholds. We plot the positions of these poles in Fig. 1, together with the poles of the full amplitude in the coupled-channel framework. The figure obviously suggests that the pole $z_1(\bar{K}N$ only) is the origin of the pole z_1, whereas $z_2(\pi\Sigma$ only) evolves to the pole z_2. This observation agrees with the qualitative behavior discussed in Ref. 11; the pole z_1 strongly couples to the $\bar{K}N$ channel and the pole z_2 to the $\pi\Sigma$ channel.

It is interesting to note that the higher energy $\bar{K}N$ channel has stronger attraction to generate a bound state, and the lower energy $\pi\Sigma$ channel shows the relatively weaker attraction, which is nevertheless strong enough to create a resonance. The appearance of the two poles in this energy region is caused by the balance of the two attractive forces. Similar pole structure is observed in the meson-meson scattering sector: σ and $f_0(980)$ resonances
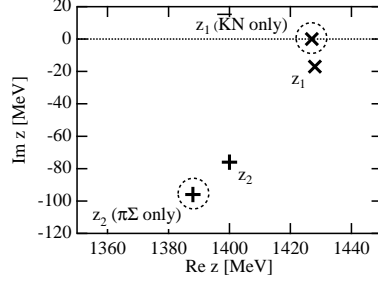
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Fig. 1.   Pole positions of the results of single-channel resummation [$z_1(\bar{K}N$ only) and $z_2(\pi\Sigma$ only)] together with the poles in the $\bar{K}N(I = 0)$ full coupled-channel amplitude (z_1 and z_2).

in $\pi\pi$-$\bar{K}K$ system (flavor structure is the same). In this sense, the physics of the lower energy pole is related to the σ meson through chiral symmetry.

4. Effective single-channel potential and K^-pp system

Keeping the structure of the $\Lambda(1405)$ in mind, we construct an effective single-channel $\bar{K}N$ interaction which incorporates the dynamics of the other channels 2-4 ($\pi\Sigma$, ηN, and $K\Xi$). We would like to obtain the solution T_{11} of Eq. (2) by solving a single-channel equation with kernel interaction V^{eff},

$$T^{\text{eff}} = V^{\text{eff}} + V^{\text{eff}} G_1 T^{\text{eff}} = T_{11}.$$

Consistency with Eq. (2) requires that V^{eff} be the sum of the bare interaction in channel 1 and the contribution from other channels (2-4), which includes iterations of one-loop terms to all orders. This method is exactly equivalent to the original coupled-channel framework, as far as the $\bar{K}N$ scattering amplitude is concerned.

In this way, the effective $\bar{K}N$ interaction V^{eff} is calculated with a chiral coupled-channel model.[9] It turns out that the $\pi\Sigma$ and other coupled channels enhance the strength of the interaction at low energy, although not by a large amount. The primary effect of the coupled channels is found in the energy dependence of the interaction kernel. By calculating the scattering amplitude, it is found that the resonance structure in the $\bar{K}N$ channel is observed at around 1420 MeV, higher than the nominal position of the $\Lambda(1405)$. The deviation of the resonance position has a large impact to the single-channel $\bar{K}N$ potential, which should be constructed so as to reproduce the scattering amplitude in $\bar{K}N$ channel. Measuring from the $\bar{K}N$ threshold (~ 1435 MeV) we find the "binding energy" of the $\Lambda(1405)$ as

~ 15 MeV, not as the nominal value of ~ 30 MeV. This shift of the binding energy apparently reduces the strength of the potential.

Next we construct an equivalent local $\bar{K}N$ potential in coordinate space. We consider an s-wave antikaon-nucleon system in nonrelativistic quantum mechanics through the Schrödinger equation with a potential $U(r, E)$. As explained in detail in Ref. 7, the local potential $U(r, E)$ has been constructed such that the scattering amplitude in coupled-channel approach is reproduced in this system. This is not an exact transformation, since it is not guaranteed that a simple local potential can reproduce the complicated coupled-channel dynamics. Nevertheless, we have constructed a complex and energy-dependent $\bar{K}N$ potential with the gaussian form of the spatial distribution, which well reproduces the coupled-channel results.

When the potential is applied to the variational calculation of K^-pp system together with the realistic NN interaction,[8] bound state solution is found with a smaller binding energy than the other calculations.[3] The main reason for the small binding is the weaker attraction of the effective $\bar{K}N$ potential. In our approach, chiral symmetry requires the strong $\pi\Sigma$ dynamics, and the attraction force to form the $\Lambda(1405)$ is divided into $\bar{K}N$ and $\pi\Sigma$ channels. As a consequence, the allotment of the $\bar{K}N$ attraction is rather small.

Present analysis focuses on the $\bar{K}NN$ component, since the $\pi\Sigma N$ channel is eliminated from the model space. For much lower energy region close to the $\pi\Sigma N$ threshold, explicit treatment of the $\pi\Sigma N$ channel would play an important role,[12] which may be related to the lower energy pole in the $\bar{K}N$-$\pi\Sigma$ amplitude. In addition, explicit treatment of ΛN channel is mandatory, in order to compare the theoretical prediction of the bound state with the experimentally observed bump structure in the ΛN spectrum[2]

5. Summary and perspective

We have derived an effective $\bar{K}N$ interaction based on chiral low energy theorem and the coupled-channel dynamics. We show that the chiral interaction leads to a strongly interacting $\pi\Sigma$-$\bar{K}N$ system, in which the $\Lambda(1405)$ is described as the $\bar{K}N$ quasibound state embedded in the *resonating $\pi\Sigma$* continuum. We construct an equivalent local potential in single $\bar{K}N$ channel, which represents the effect of coupled-channel dynamics through the imaginary part and energy dependence. As a consequence of the strong $\pi\Sigma$ dynamics, the resulting potential is less attractive than the purely phenomenological potential in the subthreshold energy range.

It is worth emphasizing that there is no direct experimental constraint

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on the $\bar{K}N$ amplitude below threshold. We have to extrapolate $\bar{K}N$ inter-action calibrated by scattering data above threshold, down to the relevant energy scale. Here we utilize the principle of chiral SU(3) symmetry in or-der to reduce the ambiguity of the extrapolation. Comprehensive study of the threshold $\bar{K}N$ data and the spectrum of the $\Lambda(1405)$ should play an important role to constraint the $\bar{K}N$ interaction below threshold.

Acknowledgments

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